# Hawking Radiation and the Unruh Effect: How Bifurcate Killing Horizons Underlie the Observer-Dependency of Thermal Radiation

Benjamin Knepper*
*Department of Physics, University of California, Berkeley, California, 94720, USA*

Professor Ori Ganor

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The discovery of black hole thermodynamics led to two fascinating results: Hawking Radiation and the Unruh Effect. We provide the QFT-in-curved-spacetime derivations for both, considering both a Bogoliubov transformation and Wick rotation approach. Both Hawking radiation and the Unruh effect dictate that observers in highly gravitational or non-inertial settings will measure thermal radiation, and we provide a comparative interpretation which unifies them through the presence of a bifurcate Killing horizon: both phenoemna boil down to a time translation symmetry between two coordinatizations generated by the bifurcate horizon.

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* bknepper@berkeley.edu

## I. INTRODUCTION

Fascinating effects can can be discovered when analyzing quantum field theory (QFT) in curved spacetime. Among them, two of the most surprising are Hawking radiation and the Unruh effect. By considering how quantum fields scatter off a black hole, Stephen Hawking calculated that black holes radiate a blackbody spectrum of particles, and therefore evaporate. William George Unruh showed that an accelerating observer in Minkowski spacetime will similarly experience a thermal bath of particles. Both effects indicate that observers in non-inertial settings will experience thermality. In this paper we derive both of these effects, and further argue that their underlying similarity is a result of a more fundamental time translation symmetry between "inertial" and "Killing" time common to both. Specifically, we show that the presence of a bifurcate Killing horizon is at the core of two approaches for calculating these effects—a Bogoliubov transformation between vacuua, and a Wick rotation—and results in the first law of black hole thermodynamics.

The paper is organized as follows. First we provide a brief overview of classical black hole thermodynamics in section II to motivate Hawking's calculation. Then we derive the Hawking radiation in section III, following Hawking's original semi-classical approach. Subsequently, we present the Unruh Effect in section IV and compare it to Hawking radiation in section V. Therein, we provide a unified interpretation in terms of "bifurcate Killing horizons." In this paper we will use units $c = \hbar = G_N = 1$ unless explicitly stated.

## II. OVERVIEW OF BLACK HOLE THERMODYNAMICS

During the 1970s, Hawking, Bekenstein and collaborators pioneered a series of discoveries that black holes exhibit behavior analogous to thermodynamic properties [1–3]. To illustrate this point, let us consider a stationary Kerr black hole which has the metric

$$ds^2 = -\Big(1-\frac{2Mr}{\rho^2}\Big)dt^2 - \frac{2Mar\sin^2\theta}{\rho^2}(dtd\phi+d\phi dt) + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\big[((r^2+a^2)^2 - a^2\Delta\sin^2\theta\big]d\phi^2 \tag{1}$$

where $\Delta(r) = r^2 - 2Mr + a^2$, $\rho^2(r,\theta) = r^2 + a^2\cos^2\theta$, and $a = J/M$ [4]. A stationary system is one that has no explicit time dependence, which is an apt choice to analyze equilibrium thermodynamic properties. Much like thermodynamic systems, a Kerr black hole is entirely characterized by set of physical parameters—mass $M$, angular momentum $J$, surface gravity $\kappa$—so we might expect there to be some correspondence with quantities such as energy, temperature, and entropy.

Indeed, one can derive an analogous "first law of black hole thermodynamics." One approach is to consider the irreducible mass of the black hole defined by $M_{irr}^2 = \frac{1}{2}(M^2 + \sqrt{M^4 - J^2})$ [4]. Computing the exact differential $\delta M_{irr}$

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega_H \delta J \tag{2}$$

where $\kappa$ is the "surface gravity" of the black hole (defined momentarily), $A$ is the area, $\Omega_H$ is the angular velocity, and $J$ is the angular momentum, as was originally derived in [5]. Noting the close correspondence between equation (2) and the first law of thermodynamics

$$dE = TdS - PdV, \tag{3}$$

we can associate $M$ with $E$, $S$ with $A$, $T$ with $\kappa/8\pi$, $P$ with $\Omega_H$, and $J$ with $V$. Equation 2 is therefore called the "first law of black hole thermodynamics," which we will return to and derive more rigorously in section V.

Equation (2) also aligns with the second law of thermodynamics in light of the "area theorem," which states that the area of the a black hole is nondecreasing [6, 7]. Since $A$ is associated with $S$, one recovers the "second law of black hole thermodynamics"

$$\delta A \geq 0 \iff \delta S \geq 0. \tag{4}$$

These results can also be extended to Kerr-Newman (rotating and charged) black holes, in which the charge acts like a chemical potential term [6, 8]. They have also been analyzed in various spacetimes, such as de Sitter [9].

The identification of temperature with surface gravity $\kappa$ will be a central focus of this paper. The surface gravity of a black hole defines the failure of a Killing parameter corresponding to an isometry (e.g. time translation symmetry) to be affine on a horizon. If a given Killing vector $\chi^\mu$ is normal to a null hypersurface $\Sigma$, then $\Sigma$ is called a *Killing horizon* of $\chi^\mu$ [4, 6]. Since on this surface $\chi^\mu \chi_\mu = 0$, the vector $\nabla^\nu(\chi^\mu \chi_\mu)$ must be normal to $\Sigma$ and therefore parallel to $\chi^\nu$ [7]. Therefore, to every Killing vector on a Killing horizon, we can associate a constant of proportionality known as the "surface gravity" $\kappa$ satisfying

$$\nabla^\nu(\chi^\mu \chi_\mu) = -2\kappa\chi^\nu \tag{5}$$

Using Killing's equation $\nabla_{(\mu}\chi_{\nu)} = 0$ and the fact that $\chi^\mu$ is normal to $\Sigma$ so $\chi_{[\mu}\nabla_\nu\chi_{\sigma]} = 0$, the surface gravity can be defined via

$$\kappa^2 = -\frac{1}{2}\Big(\nabla_\mu \chi_\nu\Big)\Big(\nabla^\mu \chi^\nu\Big). \tag{6}$$

Why physically might we expect that $\kappa$ is related to $T$? As pointed out by Wald [7], the connection is reinforced when considering the analog of the zero'th law of thermodynamics: for an object in equilibrium, the temperature must be uniform over all parts. Similarly for a stationary black hole, the surface gravity $\kappa$ will be uniform over the entire area [6]. For a spherical black hole, this follows from Birkhoff's theorem which asserts that any spherically symmetric solution to Einstein's equations will be static, and therefore have a timelike Killing vector $\chi^\mu$ that is orthogonal to a family of hypersurfaces [4]. Another qualitative motivation for this correspondence is that in thermodynamics, temperature quantifies the stubbornness of a system to give up energy [10], and the surface gravity defines the resistance of a Killing parameter to be affine.

Thus, we see that there are close mathematical correspondences between three laws of black hole and ordinary thermodynamics. If these correspondences entail equivalence, then $\kappa$ would truly yield a *physical* temperature of the black hole[1]. It was not until Hawking's infamous calculation in 1975 [12] that made the physical, quantum mechanical connection between $\kappa$ and $T$ robust.

[1] This view has been advocated by Wald [6] and recently by Almheiri et al. [11] in what they call the "central dogma:" when viewed from the outside, a black hole is as an ordinary quantum mechanical system, obeying the laws of thermodynamics.

## III. HAWKING RADIATION

Hawking discovered that a black hole does not only absorb particles, but also emits particles as if it were a hot object with temperature $T_H = \kappa/2\pi$ [12]. We now present Hawking's original "semi-classical" derivation of this result, which relies on analyzing quantum field theory in classical curved spacetime. The perspective Hawking adopts is to view the interaction between the black hole and the quantum fields as a *scattering problem*. In particular, the overall goal is to calculate a quantity analogous to the S-matrix, relating the quantum field vacuum at very early times to the vacuum at very late times after scattering off the black hole.

Specifically, we start by considering a static Schwarzschild black hole formed by stellar collapse, with metric

$$ds^2 = -\Big(1 - \frac{r_s}{r}\Big)dt^2 + \Big(1 - \frac{r_s}{r}\Big)^{-1}dr^2 \tag{7}$$

where $r_s = 2M$ is the Schwarzschild radius and where we have neglected the $r^2 d\Omega^2$ angular component. Around this classical black hole, we introduce a quantum scalar field $\phi$ which satisfies the Klein-Gordon equation

$$(\Box - m^2)\phi(r,t) = 0 \tag{8}$$

where $\Box = \nabla^\mu\nabla_\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$ is the D'Alambert operator for the Schwarzschild metric $g^{\mu\nu}$, and $g = \det\{g_{\mu\nu}\}$ [13]. For simplicity, we will consider a massless scalar field (however these results have been shown to hold for the massive case and for Dirac fields; see [14]).

Normally if we were in flat spacetime, we would be able to unambiguously define initial conditions to determine the positive frequency modes of this field by the constraint

$$\partial_t\phi^+ = -i\omega\phi^+, \quad \omega > 0. \tag{9}$$

However, in curved spacetime, there is no covariant notion of positive frequencies. This is because the condition (9) relies on a timelike Killing vector $\partial_t$, but its existence is not guaranteed in curved spacetime and moreover there could be many possible hypersurfaces on which to define this Killing vector [4, 12, 15]. In the case of the Schwarzschild metric (7), there is no $t$ dependence so there will be a timelike Killing vector in line with Birkhoff's theorem, but we need to specify on which null surface it is defined. This time translation symmetry will become crucial to the present analysis.

### A. Schwarzschild and Kruskal Coordinates

To draw out this effect more explicitly, let us consider two possible coordinatizations of the Schwarzschild black hole: the "Schwarzschild" and "Kruskal" coordinates. For $r \to r_s$, the metric (7) diverges so we can introduce a coordinate system better equipped to analyze causal structure of geodesics approaching the horizon. To do so, we consider the behavior of null rays by setting $ds^2 = 0$ and solving the resulting differential equation

$$\frac{\mathrm{d}t}{\mathrm{d}r} = \pm\Big(1 - \frac{r}{r_s}\Big)^{-1} \tag{10}$$

to obtain $t = \pm r^*$, where $r^*$ is the "tortoise coordinate" given by [4]

$$r^* = r + r_s \log\left(\frac{r}{r_s} - 1\right) \tag{11}$$

up to a constant depending on $r_s$ and a reference radius $r_0$. Effectively, this calculation slows down the time coordinate of the null ray so that we can "catch up" to it to see how it behaves as it approaches the event horizon before it diverges. One can then define the corresponding coordinates of a null ray in this parameterization,

$$v = t + r^* \tag{12}$$

$$u = t - r^* \tag{13}$$

where $u$ is the "retarded" time of an outgoing null ray moving towards $r \to \infty$ and $v$ is the "advanced time" of an ingoing ray moving towards $r \to 0$. We will refer to $(u, v)$ as the "Schwarzschild" coordinates. The metric in the Schwarzschild coordiantes becomes

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) du dv. \tag{14}$$

While the Schwarzschild coordinates fix the divergence as $r \to r_s$, the metric (14) vanishes and equivalently the horizon becomes effectively infinitely far away at either $v = -\infty$ or $u = \infty$ due to the divergence of the tortoise coordinate. We can therefore define a further global coordinate system to better analyze the transition of geodesics across the horizon. We do this by exponentiating,

$$V = e^{v/2r_s} = e^{(r^*+t)/2r_s} \tag{15}$$

$$U = -e^{-u/2r_s} = -e^{(r^*-t)/2r_s} \tag{16}$$

where $(U, V)$ are the globally extended version of null coordinates $(u, v)$ known as the "Kruskal coordinates" [4], which give a well-behaved metric at $r_s$ of the form

$$ds^2 = -\frac{16M^3}{r} \exp\left(-\frac{r}{r_s}\right) dU dV \tag{17}$$

The Penrose diagram for a maximally extended Schwarzschild black hole is then obtained from a conformal transformation on these Kruskall coordinates as shown in figure 1 (a). The Schwarzschild coordinates are defined only in region I of the Penrose diagram since they are related to $U, V$ by a logarithm [4, 6]. In the case of gravitational collapse, only the region outside of the matter shell in region I has a physically relevant coordinatization (figure 1 (b)).

### B. Bogoliubov Transformation

Now, returning to field $\phi$, we can perform two equivalent mode decompositions corresponding to the field state either before or after the black hole scattering:

$$\begin{aligned} \phi &= \sum_i f_i a_i + f_i^* a_i^\dagger \\ &= \sum_i p_i b_i + p_i^* b_i^\dagger + q_i c_i + q_i^* c_i^\dagger. \end{aligned} \tag{18}$$

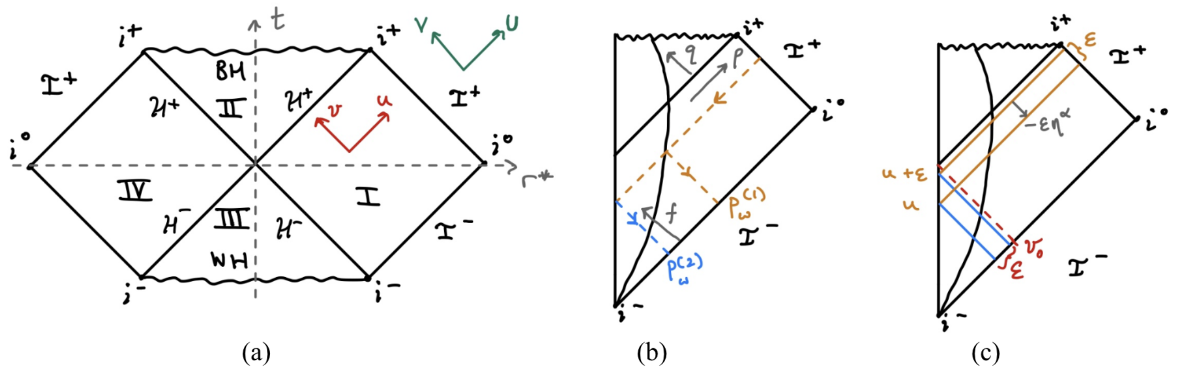



Figure 1: Penrose diagrams of a Schwarzschild black hole (BH). (a) Full Penrose diagram from extended Kruskal coordinates. (b) BH formed by stellar collapse. Shown in grey are the modes $f, p, q$. Shown in dotted lines are the modes $p_\omega^{(1)}$ and $p_\omega^{(2)}$. (c) Schematic of parallel transport procedure for calculating $p_\omega^{(2)}$.

The $\{f_i\}$ modes have positive frequencies with respect to $\mathcal{I}^-$ and represent null rays ingoing towards the black hole from past null infinity. The natural affine time parameter on $\mathcal{I}^-$ is the advanced Schwarzschild time $v$ so we can define the spherically symmetric positive frequency ingoing modes as

$$f_\omega = \frac{1}{4\pi\sqrt{\omega}}\frac{e^{-i\omega v}}{r}. \tag{19}$$

The modes $\{p_i\}$ have positive frequencies with respect to $\mathcal{I}^+$ and represent outgoing particles from the black hole towards future null infinity. In this case, the natural affine parameter is the advanced Schwarzschild time $u$ so we can similarly define outgoing positive frequency modes by

$$p_w = \frac{1}{4\pi\sqrt{\omega}}\frac{e^{-i\omega u}}{r}. \tag{20}$$

Lastly, the modes $\{q_i\}$ represent particles falling into the black hole so to avoid discussing phenomena inside the black hole, we leave the positive frequency definition unspecified, but declare that they never intersect $\mathcal{I}^-$ or $\mathcal{I}^+$. The mode operators satisfy the usual commutation relations: $\left[a_i, a_j^\dagger\right] = \left[b_i, b_j^\dagger\right] = \left[c_i, c_j^\dagger\right] = \delta_{ij}$ [12, 16, 17]. From the two decompositions of $\phi$ in (18), one can define two different vacuua which we call the "in" and "out" vacuum,

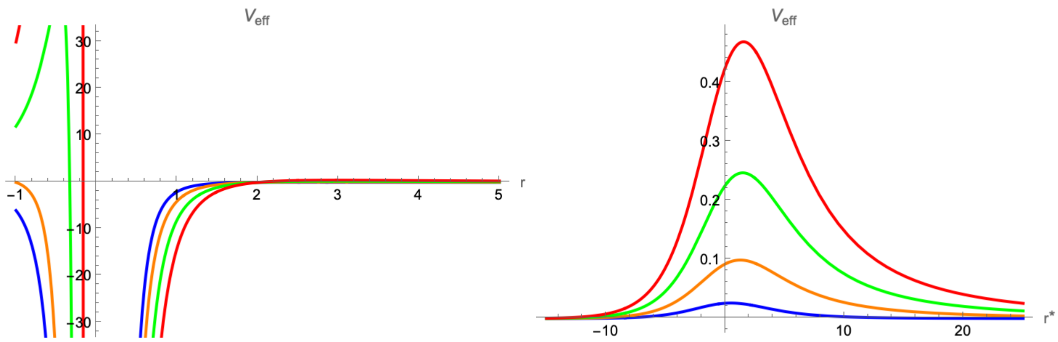
$$a_i \left|0\right\rangle_{in} = 0, \qquad b_i \left|0\right\rangle_{out} = c_i \left|0\right\rangle_{out} = 0. \tag{21}$$

The Schwarzschild metric provides an effective scattering potential between these two vacuua. This can be seen by considering the action $I$ of $\phi$ in a Schwarzschild spacetime, which in terms of the tortoise coordinate $r^*$ becomes

$$I = -\frac{1}{2}\int d^4x\sqrt{g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \int dtdr^*\frac{1}{2}\Big(\frac{\mathrm{d}(\phi r)}{\mathrm{d}t}\Big)^2 - \Big(\frac{\mathrm{d}(\phi r)}{\mathrm{d}r^*}\Big)^2 - V_{\text{eff}}(r) \tag{22}$$

where

$$V_{\text{eff}}(r) = \Big(1 - \frac{r_s}{r}\Big)\Big(\frac{l(l+1)}{r^2} + \frac{r_s}{r^3}\Big). \tag{23}$$




(a) $V_{\rm eff}(r)$ (b) $V_{\rm eff}(r^*)$

Figure 2: $V_{\rm eff}$ for various angular momenta: $l = 0$ (blue), $l = 1$ (orange), $l = 2$ (green), $l = 3$(red), with $r_s = 2, G = M = 1$. (a) Parametrization via $r$ exhibits an expected divergence as $r \to 0$. (b) Parametrization via the tortoise coordinate $r^*$ smoothly extends the potential past $r^* \to 0$.

and $l$ is the angular momentum of a mode of the field [18]. Figure 2 shows Mathematica plots of $V_{\rm eff}$ as a function of $r$ and $r^*$. An incoming mode from $\mathcal{I}^-$ with energy less than $V_{\rm eff}(r^*_{\rm max})$ can either be reflected by the hump in figure 2b or tunnel through it into the black hole interior.

In the S-matrix formulation of this scattering problem, we only need consider the transition amplitude between $|0\rangle_{in}$ at $\mathcal{I}^-$ and $|0\rangle_{out}$ at $\mathcal{I}^+$, where an asymptotic observer could measure the effects of the scattering. Therefore, only the $f$ and $p$ modes of (18) will become relevant to derive thermodynamic properties of the black hole when viewing it from the outside. The goal is to now relate the two vacuua $|0\rangle_{in}$ and $|0\rangle_{out}$ by relating the $a$ and $b$ mode operators.

To do so, we first note that as a result of the scattering process, we should be able to express the outgoing modes as linear combinations of the ingoing modes

$$p_i = \sum_j \alpha_{ij} f_i + \beta_{ij} f_j^*. \tag{24}$$

We can derive a constraint on these coefficients $\alpha_{ij}$ and $\beta_{ij}$ by considering orthogonality conditions required by the "Klein Gordon (KG) norm." For any two scalar field components $\phi_1, \phi_2$ with initial conditions determined on a Cauchy hypersurface $\Sigma$, the KG norm defines an inner product of the form

$$(\phi_1, \phi_2) = -i \int_\Sigma d\Sigma^\mu \Big(\phi_1 \partial_\mu \phi_2^* - \phi_2^* \partial_\mu \phi_1\Big) \tag{25}$$

which, for a field $\phi$ decomposed into components $\{\phi_i\}$, enforces the following orthogonality conditions [4, 6, 17]

$$(\phi_i, \phi_j) = -(\phi_i^*, \phi_j^*) = \delta_{ij}, \quad (\phi_i, \phi_j^*) = 0. \tag{26}$$

Applying the orthogonality conditions to $\phi_i = f_i$ or $\phi_i = p_i$ and using the form (24) for $p_i$, we see that $\alpha_{ij}$ and $\beta_{ij}$ must satisfy the following relations

$$\alpha_{ij} = (p_i, f_j), \quad \beta_{ij} = -(p_i, f_j^*) \tag{27}$$

$$\sum_j |\alpha_{ij}|^2 - |\beta_{ij}|^2 = \delta_{ij}. \tag{28}$$

Moreover, the same orthogonality conditions can be used to define a similar linear combination as we had before but between the "out" and "in" mode operators $b_i$ and $a_i$,

$$b_i = \sum_j \alpha_{ij}^* a_j - \beta_{ij} a_j^\dagger. \tag{29}$$

A relation such as (24) between different field modes satisfying equations (27-29) is known as a "Bogoliubov transformation" [4, 6, 17, 19].

How might we determine the alleged temperature of the black hole in this scattering formalism? The insight by Hawking was to calculate the expected number of outgoing particles $N_{b_i} = b_i^\dagger b_i$ with respect to the ingoing vacuum using (29):

$$\begin{aligned}\langle 0_{in}|b^\dagger b|0_{in}\rangle &= \sum_j \langle 0_{in}|(\alpha_{ij}a_j^\dagger - \beta_{ij}^* a_j)(\alpha_{ij}^* a_j - \beta_{ij} a_j^\dagger)|0_{in}\rangle \\ &= \sum_j |\beta_{ij}|^2 \end{aligned} \tag{30}$$

since the only non-vanishing term comes from $\langle 0_{in}||\beta_{ij}|^2 a_j a_j^\dagger|0_{in}\rangle$. Evidently, this Bogoliubov transformation between $|0\rangle_{in}$ and $|0\rangle_{out}$ is at the core of Hawking's derivation. While tedious, it is in this calculation of the Bogoliubov coefficient $\beta_{ij}$ where the surface gravity $\kappa$ directly makes its appearance.

### C. The Result: Calculation of $\beta_{ij}$

To calculate $\alpha_{ij}$ and $\beta_{ij}$ we need to use equation (27), which requires an explicit form of $p_i$. To avoid analyzing interior modes of the black hole, Hawking uses a clever trick of considering the scattering process in reverse [2]: we consider a mode $p_\omega$ with frequency $\omega$ propagating backwards from $\mathcal{I}^+$ towards the black hole, which will have a two components $p_\omega = p_\omega^{(1)} + p_\omega^{(2)}$. The $p_\omega^{(1)}$ component gets reflected by the Schwarzschild effective potential and $p_\omega^{(2)}$ tunnels through it, both ending up at $\mathcal{I}^-$ as shown in figure 1 (b) [12].

The component $p_\omega^{(1)}$ will elastically scatter and therefore contribute $\delta(\omega - \omega')$ to the $\alpha$ component of $p_\omega$ in (24). To analyze the second component, we consider two neighboring $p_\omega^{(2)}$ modes at times $u$ and $u + \epsilon$ for small $\epsilon > 0$. We set $u$ to be very close to the horizon $\mathcal{H}^+$, which corresponds to its reflected component on $\mathcal{I}^-$ being very close to $v_0$, the time after which any ingoing null ray falls into the black hole (see figure 1 (c)). There will be a normalized Killing vector $\eta^\alpha$ orthogonal to the horizon which connects these two rays via $-\epsilon\eta^\alpha$. As a proxy for considering the effect of the scattering location $r = 0$ on $-\epsilon\eta^\alpha$, we can instead imagine that we are in the global Kruskal extension of the Penrose diagram and parallel transport $-\epsilon\eta^\alpha$ along the horizon back to where $\mathcal{H}^+$ intersects $\mathcal{H}^-$ at $V = U = 0$. Let $\lambda$ be the affine parameter on $\mathcal{H}^-$. This parallel transport trick allows us to identify $\frac{\mathrm{d}x(\lambda)^\alpha}{\mathrm{d}\lambda} = \eta^\alpha$ for a geodesic $x(\lambda)^\alpha$ and relate the retarded Schwarzschild time $u$ to $\lambda$ via

$$\frac{\mathrm{d}u}{\mathrm{d}\lambda} = \frac{-r_s}{\lambda} = -\frac{1}{\kappa\lambda} \tag{31}$$

[2] This point of view is analogous to viewing a scattered electron as a positron moving backwards in time.

where the factor of $-r_s$ comes from (16) and for a Schwarzschild black hole $\kappa = \frac{1}{2M} = \frac{1}{r_s}$. Solving this equation yields $u = -\frac{1}{\kappa}\ln(\lambda)$ as expected. Therefore, deviating the $u+\epsilon$ geodesic to $u$ along $\mathcal{H}^-$ will result in $\lambda = \epsilon$,

$$u = -\frac{1}{\kappa}\ln(\epsilon). \tag{32}$$

These scattered null rays near the horizon will be highly blue-shifted, which can be seen from two perspectives: 1) near the horizon, $u = \infty$ as discussed in section III A, so the geodesics $u+\epsilon$ will pile up [12]; 2) in the scattering picture of figure 2, the geodesics near the horizon will undergo many internal reflections before tunneling back out out of the matter shell. Consequently, we can use a geometric optics approximation to trace back the rays to $\mathcal{I}^-$, which allows us to claim that the geodesic deviation distance in $u$ time is preserved in $v$ time: $v_0 - v = \epsilon$. As a result, in advanced time coordinates $p_\omega^{(2)}$ will be of the form

$$p_\omega^{(2)} \approx \exp\Big(-i\frac{\omega}{\kappa}\ln(v - v_0)\Big). \tag{33}$$

All together, the contributions of $p_\omega^{(1)}$ and $p_\omega^{(2)}$ make the full $p_\omega$ mode in equation (24) of the form

$$\frac{1}{\sqrt{\omega}r}e^{-i\omega\kappa^{-1}\ln(v_0-v)} = \sum_{\omega'}\alpha_{\omega\omega'}\frac{1}{\sqrt{\omega'}r}e^{-i\omega' v} + \beta_{\omega\omega'}\frac{1}{\sqrt{\omega'}r}e^{i\omega' v} \tag{34}$$

where we have used the frequency-space representations (19 - 20) of $f_i$.

Then to isolate $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$, we can Fourier transform the above equation and compare to the Klein Gordon norm forms (27). The resulting form is messy so will be omitted to conserve space, but is explicitly calculated out in [17]. The result is that $\beta_{ij}$ and $\alpha_{ij}$ are related by

$$\sum_j |\alpha_{ij}|^2 = e^{2\pi\omega\kappa^{-1}}\sum_j |\beta_{ij}|^2. \tag{35}$$

Substituting equation (35) into (28) and re-arranging, the expected particle number (30) is

$$\langle 0_{in}|b^\dagger b|0_{in}\rangle = \sum_j |\beta_{ij}|^2 = \frac{\delta_{ij}}{e^{2\pi\omega\kappa^{-1}} - 1} \tag{36}$$

which is exactly the result for a thermal blackbody spectrum with temperature $T_H = \kappa/2\pi$ [12, 16]. This is the Hawking temperature.

## IV. THE UNRUH EFFECT

Interestingly, there is an analagous effect to Hawking radiation in flat spacetime known as the "Unruh Effect": an accelerating observer in Minkowski will experience a bath of thermal radiation [14]. We now discuss the derivation of this result.

To start, consider standard Minkowski space,

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{37}$$

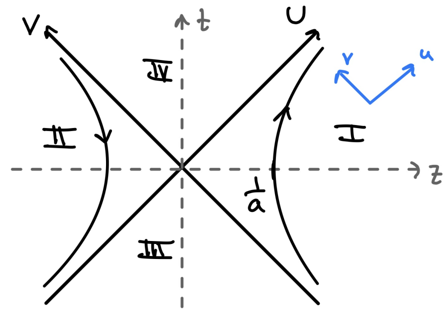



Figure 3: Spacetime diagram of Minkowski space. The "left" and "right" Rindler wedges correspond to regions I and II.

and consider including a uniformly accelerating observer with acceleration in the MRCF given by $a^\mu = (0, 0, 0, a)$. The proper coordinates for this accelerating observer's geodesics will just be given by the familiar boosted coordinate transformation, parametrized by proper time $\tau$

$$t(\tau) = \rho \sinh(\eta(\tau)), \quad z(\tau) = \rho \cosh(\eta(\tau)) \tag{38}$$

where $\rho = 1/a$ and $\eta = a\tau$, which are known as "Rindler coordinates." The metric in Rindler coordinates becomes [15]

$$ds^2 = -\rho^2 d\eta^2 + d\rho^2 + dx^2 + dy^2. \tag{39}$$

We can further define null coordinates $(u, v)$ analogous to the Schwarzschild coordinates by [14]

$$v = \tau + \ln(\rho), \quad u = \tau - \ln(\rho) \tag{40}$$

and a global extension of these coordiantes $(U, V)$ analogous to the Kruskal coordinates via [15]

$$V = t + z = \frac{1}{a} e^{av}, \quad U = t - z = -\frac{1}{a} e^{-au}. \tag{41}$$

Like in the Hawking case, $(u, v)$ are defined on only the left or right Rindler wedges ($z^s > t^2$) and $(U, V)$ can be extended across either, as shown in figure 3 [15].

Then, as in the Hawking calculation, we can express a free scalar field in the presence of this accelerating observer in terms of two mode expansions,

$$\phi = \sum_i f_i a_i + \bar{f}_i a_i^\dagger \tag{42}$$

$$= \sum_i g_{l,i} b_{l,i} + g_{l,i}^* b_{l,i}^\dagger + g_{r,i} b_{r,i} + g_{r,i}^* b_{r,i}^\dagger. \tag{43}$$

The expression (42) is for the Minkowski modes and (43) is for the Rindler modes, where $\{g_{l,i}\}$ denote modes on the left Rindler wedge and $\{g_{r,i}\}$ denote modes on the right Rindler wedge. As before, these two decompositions define a Minkowski (M) and Rindler (R) vacuum via

$$a_i \left|0\right\rangle_M = 0, \qquad b_{l,i} \left|0\right\rangle_R = b_{r,i} \left|0\right\rangle_R = 0 \tag{44}$$

We could then proceed by calculating the Bogoliubov coefficients between for the transformation between $|0\rangle_M$ and $|0\rangle_R$ as in the Hawking radiation derivation [15]. Unruh, however, used a shortcut which will be illustrative for the analysis in section V. Unruh considered globally rotating the coordinates from the right wedge to the left wedge by $\eta \to \eta - i\pi$ [14, 20]. This analytic continuation of the time coordinate is an example of what is called a "Wick rotation." It transforms the positive frequency mode coefficients as

$$g_{l,i} = e^{-i\omega\eta} \to e^{-\omega\pi} e^{-i\omega\eta} = e^{-\omega\pi} g^*_{r,i} \tag{45}$$

since $g_{r,i}$ are positive frequency modes with respect to $\eta$ in the right wedge and $g_{l,i}$ are negative frequency modes since $\eta$ decreases towards the future in the left wedge [20]. Therefore, we can calculate the normalized positive frequency modes using the Klein Gordon norm to arrive at

$$h_{l,i} = \sqrt{\frac{e^{\pi\omega}}{2\sinh(\pi\omega)}}\left(g_{l,i} + e^{-\pi\omega} g^*_{r,i}\right) \tag{46}$$

Since we also could have performed the rotation $\eta \to \eta + i\pi$ from the left wedge to the right wedge, we can get an analogous expression $h_{r,i}$. In terms of these newly defined positive frequency modes, the field takes the form

$$\phi = \sum_i h_{l,i} c_{l,i} + h^*_{l,i} c^\dagger_{l,i} + h_{r,i} c_{r,i} + h^*_{r,i} c^\dagger_{r,i} \tag{47}$$

Applying the Klein Gordon normalization conditions (26) to the two field representations (47) and (43) also yields the following relation between mode operators

$$b_i = \sqrt{\frac{e^{\pi\omega}}{2\sinh(\pi\omega)}}\left(c_i + e^{-\pi\omega} c^\dagger_i\right) \tag{48}$$

Importantly, by deriving these globally defined positive frequency coordinates, we can now relate now relate to the $h_i$ frequencies to the original Minkowski frequencies, meaning the mode operators $c_i$ are eigenstates of the Minkowski vacuum: $c_{l,i}|0\rangle_M = c_{r,i}|0\rangle_M = 0$.

As in the Hawking derivation, we can then calculate the expected particle number of the Rindler observer in the right wedge with respect to the Minkowski vacuum using (48),

$$\begin{aligned}\langle 0_M| b^\dagger_{r,i} b_{r,i} |0_M\rangle &= \frac{e^{\pi\omega a^{-1}}}{2\sinh \pi\omega a^{-1}} \langle 0_M| b_{r,i} b^\dagger_{r,i} |0_M\rangle \\ &= \frac{1}{e^{2\pi\omega a^{-1}} - 1}\delta(0)\end{aligned} \tag{49}$$

which corresponds to a thermal distribution with temperature $T_U = \frac{a}{2\pi}$. This is the Unruh temperature.

This suggests that we can construct a purified Minkowski vacuum across both left and right wedges of the form

$$|0\rangle_M = \prod_i \sum_{n=0}^{\infty} e^{-n\pi\omega_i a^{-1}} |n_{l,i}\rangle |n_{r,i}\rangle \tag{50}$$

which upon tracing out either the left or right side yields a Gibbs thermal density matrix with temperature $T_U$ [7].

## V. ANALYSIS AND COMPARISON OF HAWKING AND UNRUH EFFECTS

Both the Unruh Effect and Hawking effect predict that with respect to an initial vacuum, the effects of acceleration or gravity would result in a thermal spectrum of particles. Where Hawking's derivation related ingoing and outgoing coordinates from black hole scattering, Unruh's calculation relates left and right Rindler wedge coordinates to positive frequencies defined from an accelerating frame. This mathematical resemblance seems like the principle of equivalence at work, and indeed this can be shown by zooming into the horizon of a black hole.

Expanding around $r = r_s + \epsilon^2$ to first order in $\epsilon^2/r_s \ll 1$, the Schwarzschild metric becomes

$$ds^2 = 4r_s\Big(-\frac{\epsilon^2}{4r_s^2}dt^2 + d\epsilon^2\Big) \tag{51}$$

which is equivalent to the Rindler metric (39) up to a global scaling. From the Wick rotation method discussed in the previous section, we can infer that in this region the time coordinate must be periodic, $t \to t + 2\pi i$.

In QFT, it is a standard result that using a similar Wick rotation sending $t \to i\beta$ converts a propagator into a thermal Greens function which will be periodic in $\beta$,

$$\mathrm{tr}[\exp(-iHt)] \to \mathrm{tr}[\exp(-\beta H)] = Z(\beta) \tag{52}$$

where $\beta$ is the inverse temperature of thermal partition function $Z(\beta)$. Applying such a Wick $t \to i\tau$ rotation to the full Schwarzschild metric will convert it from Lorentzian to Euclidean signature,

$$ds^2 = \Big(1 - \frac{r_s}{r}\Big)d\tau^2 + \Big(1 - \frac{r_s}{r}\Big)^{-1}dr^2 \tag{53}$$

To avoid a conical singularity at $r = r_s$, we match the periodicity conditions of the thermal Green's function to the Rindler metric and conclude that $\beta$ must be given by $\beta = 4\pi r_s$. This results in a temperature [11, 21]

$$T = \beta^{-1} = \frac{\kappa}{4\pi} \tag{54}$$

which agrees with the Hawking temperature $T_H$ up to a factor of 2 which stems from our approximation of the metric near the horizon. For a more in depth analysis of the relation between Euclidean signature, Feynman propagators, and thermal Green's functions, see [22–24].

Thus, for an observer very close to the horizon, the Hawking and Unruh Effects are the same [11, 25]. This is a confirmation of the principle of equivalence. However for asymptotic regions from the horizon, the Unruh effect and Hawking effect are distinct and describe two different quantum states [7, 25]. This leads us to ask: what is the underlying principle which might relate the Unruh effect to the Hawking effect even for asymptotic observers, and what relates the Bogoliubov transformation to the Wick rotation approach?

A closer analysis reveals that the underlying similarity arises because both effects involve a *bifurcate Killing horizon*. A bifurcate killing horizon divides spacetime into four wedges I, II, III, IV, and occurs whenever there are two null surfaces $\mathcal{H}^+, \mathcal{H}^-$ which intersect on a space-like "bifurcation surface" $\mathcal{S}$ [7, 25]. This bifurcate Killing horizon introduces two possible conformally related time coordinates, and the observed thermal spectra in both the Hawking and Unruh effects boils down to a choice between the two coordinates. In the case of black hole evaporation, this is a choice between the Schwarzschild coordinates $(u, v)$ and Kruskal coordinates $(U, V)$. In the case of an accelerating observer in Minkowski, this is a choice between Minkowski coordinates $(u, v)$ or Rindler coordinates $(U, V)$. Wald calls these two types of coordinates in general "Killing times" or "inertial times," respectively [7].

The relation between Killing and inertial times of a bifurcate Killing horizon always follows the same general form, which we now show. The Killing equation $\nabla_{(\mu}\chi_{\nu)} = 0$ allows us to re-write the the left hand side of equation (5) as $-2\chi^\mu\nabla_\mu\chi^\nu$ so the equation becomes

$$\chi^\mu\nabla_\mu\chi^\nu = \kappa\chi^\nu \tag{55}$$

which is just the geodesic equation

$$0 = \frac{\mathrm{d}^2x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{\mathrm{d}x^\rho}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\sigma}{\mathrm{d}\lambda} \tag{56}$$

with Scharzschild Christoffel symbols written in a non-affine parametrization [7]. Then, if $v$ is a Killing parameter on $\mathcal{H}^+$ and $u$ is a Killing parameter on $\mathcal{H}^-$, we can define inertial (i.e. affine) parameters $(V, U)$ on $\mathcal{H}^+$ and $\mathcal{H}^-$, respectively, such that

$$v = \frac{1}{\kappa}\ln|V| \qquad \Longleftrightarrow \qquad V = \exp(\kappa v) \tag{57}$$

$$u = -\frac{1}{\kappa}\ln|U| \qquad \Longleftrightarrow \qquad U = \exp(-\kappa u) \tag{58}$$

which is the general form for both the Kruskal coordinates (16 - 15) where $\kappa = 1/2r_s$, and for the Rindler coordinates (41) after performing the Wick rotation. We now show that this bifurcate Killing horizon time translation symmetry is at the core of the Bogoulibouv transformation calculation approach, the Wick rotation approach, and that it provides a derivation for the first law of black hole thermodynamics.

### A. Application 1: Bogoliubov transformation between vacuua

There are three physically significant and distinct vacuua that one can consider in the case of an evaporating black hole: the Boulware vacuum $|0_B\rangle$, the Hartle-Hawking vacuum $|0_{HH}\rangle$, and the Unruh vacuum $|0_U\rangle$ [7, 20]. They can be heuristically written in terms of Kruskal coordinates $K$ and Schwarzschild coordinates $S$ as [19]

$$|0_B\rangle_{in} = |0_{\mathcal{I}^-}\rangle_S\,|0_{\mathcal{H}^-}\rangle_S\,, \quad |0_B\rangle_{out} = |0_{\mathcal{I}^+}\rangle_S\,|0_{\mathcal{H}^+}\rangle_S \tag{59}$$

$$|0_{HH}\rangle_{in} = |0_{\mathcal{I}^-}\rangle_K\,|0_{\mathcal{H}^-}\rangle_K\,, \quad |0_{HH}\rangle_{out} = |0_{\mathcal{I}^+}\rangle_K\,|0_{\mathcal{H}^+}\rangle_K \tag{60}$$

$$|0_U\rangle_{in} = |0_{\mathcal{I}^-}\rangle_S\,|0_{\mathcal{H}^-}\rangle_K\,, \quad |0_U\rangle_{out} = |0_{\mathcal{I}^+}\rangle_S\,|0_{\mathcal{H}^+}\rangle_K \tag{61}$$

where $\pm$ refers to future or past, $\mathcal{H}^\pm$ refers to the future or past horizon of the bifurcate Killing horizon, and $\mathcal{I}^\pm$ refers to future or past null infinity.

The key point is that the physically relevant vacuum which Hawking implicitly uses in his calcualtion is the Unruh vacuum $|0_U\rangle$. Specifically, in his trick of parallel transporting the Killing vector $\eta^\alpha$ from $\mathcal{H}^+$ onto $\mathcal{H}^+$ to arrive at equation (31), Hawking implicitly sets $\lambda = U$. At its core, the Bogoliubov transformation rests on this step of transforming to global coordinates and choosing $K$ on $\mathcal{H}^\pm$ and S on $\mathcal{I}^\pm$, which amounts to choosing the Unruh vacuum (61). As a result, the state is a thermal ensemble for outgoing modes in $K$ coordinates from the horizon towards $\mathcal{I}^+$ and a pure vacuum for ingoing modes in $S$ coordinates towards the horizon from $\mathcal{I}^-$ [7].

Moreover, if an observer were to measure particle number with respect to the Boulware vacuum $|0_B\rangle$, they would not see a thermal spectrum [20, 26]. This is because for an asymptotic observer at $\mathcal{I}^+$, the "in" vacuum would be an eigenstate of the "out" mode operators given that the positive frequencies are defined with respect to the same times in $S$ coordinates, so that $\langle 0_B|b^\dagger_{B,out} b_{B,out}|0_B\rangle_{in} = 0$.

### B. Application 2: Wick Rotation

An alternative approach to deriving both the Hawking and Unruh effects is to use a Wick rotation, which makes the time coordinate imaginary. In the case of an accelerating observer, this analytic continuation of $\eta \to \eta + i\pi$, is enabled by the bifurcate Killing horizon as it effectively redefines the left or right wedge time coordinates $(u, v)$ in terms of global time coordinates $(U, V)$.

A more general Wick rotation transforms $\eta \to i\theta$, and under the Euclidean interpretation this angle $\theta$ becomes the new time. Hence, the Rindler coordinates become

$$t = \rho\cos(\theta), \quad x = \rho\sin(\theta) \tag{62}$$

which represents an observer at constant $\rho$ moving in a circle of length $2\pi\rho$ in Euclidean space [11, 20]. Multiples of $\theta = \pi n, n \in \mathbb{Z}$ therefore correspond to a rotation in Lorentzian coordinates between $(u, v)$ on the left or right wedges which are defined with respect to global coordinates $(U, V)$, as discussed in section IV.

### C. Application 3: First Law of Black Hole Thermodynamics

In addition, following Wald [7] we can show that the first law of black hole thermodynamics follows from the presence a bifurcate Killing horizon. Consider a Kerr black hole as in section II. In this metric (1), there are two conserved quantities—time and angular momentum—so the general Killing vector of these isometries $\xi^\mu$ is a linear combination of the form

$$\chi^\mu = \xi^\mu + \Omega\varphi^\mu \tag{63}$$

where $\chi^\mu = \frac{\partial}{\partial v^\mu}$ is given by the "Killing" time coordinate $v$ [6].

We can invoke a focusing analysis similar to Hawking and Penrose's singularity theorems to get an expression for the area of the horizon. In particular, the expansion $\theta$ of null geodesics parametrized by $\lambda$ on the black hole horizon $\mathcal{H}^+$ is related to the horizon area by [7]

$$\theta = \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}\lambda}. \tag{64}$$

Following the bifurcate Killing horizon argument of this section, the natural choice of affine parameter is the "inertial time" $V$ so we set $\lambda = V$. Applying the geodesic deviation equation to (64), which expresses the relative acceleration between a family of geodesics (or similarly the rate of change of $\theta$ with respect to the $V$), the result is the Raychaduri equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}V} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - R_{\mu\nu}K^\mu K^\nu \tag{65}$$

where $\sigma_{\mu\nu}$ is a shear term for the geodesic deviation, $R_{\mu\nu}$ is the Ricci curvature tensor, and $K^\mu = \frac{\partial}{\partial V^\mu}$ is the Killing vector defined with respect to $V$ [6, 7]. Then using Einstein's equation

$$R_{\mu\nu} = 8\pi\Big(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\Big), \tag{66}$$

the Raychaduri equation can be re-written in terms of the stress-energy tensor as [4, 7]

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - 8\pi\Big(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\Big)K^\mu K^\nu. \tag{67}$$

We now consider perturbing the black hole slightly by dumping a small amount of matter $\delta T_{\mu\nu}$ into it. The resulting change in black hole mass and angular momentum is given by integrating this stress-energy tensor along with the appropriate Killing fields,

$$\delta M = \int_0^\infty dV \int d^2S\delta T_{\mu\nu}\xi^\mu K^\nu \tag{68}$$

$$\delta J = \int_0^\infty dV \int d^2S\delta T_{\mu\nu}\Omega\varphi^\mu K^\nu \tag{69}$$

where $S$ is a horizon cross-section [7, 8]. To first order in $\delta T_{\mu\nu}$ we can neglect any quadratic terms in the Raychaduri equation so it becomes

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = -8\pi\delta T_{\mu\nu}K^\mu K^\nu. \tag{70}$$

We now want to write $K^\mu$ in a more suggestive form. Using the relation between "Killing" and "inertial" time (57), we can write $\partial V^\mu = \kappa V\partial v^\mu$. Therefore, we can relate $K^\mu$ to $\chi^\mu$ via

$$K^\mu = \frac{\partial}{\partial V^\mu} = \frac{1}{\kappa V}\frac{\partial}{\partial v^\mu} = \frac{1}{\kappa V}\chi^\mu = \frac{1}{\kappa V}(\xi^\mu + \Omega\varphi^\mu) \tag{71}$$

where we have used (63) in the final equality. Then multiplying (70) through by $\kappa V$ and integrating, we obtain

$$\kappa\int_0^\infty dV\int d^2SV\frac{\mathrm{d}\theta}{\mathrm{d}V} = -8\pi\int_0^\infty dV\int d^2S\delta T_{\mu\nu}(\xi^\mu + \Omega\varphi^\mu)K^\nu \tag{72}$$

The left hand side can be integrated by parts

$$\kappa\int_0^\infty dV\int d^2SV\frac{\mathrm{d}\theta}{\mathrm{d}V} = \int d^2S\theta V|_0^\infty - \int d^2S\int_0^\infty dV\theta = \delta A \tag{73}$$

where the first term vanishes since $V = 0$ at $S = 0$ and the expansion $\theta \to 0$ as $V \to \infty$. The right hand side of (72) gives the integrals (68-69). All together, this gives the desired result

$$\kappa \delta A = 8\pi(\delta M - \Omega \delta J). \tag{74}$$

Evidently, the first law of black hole thermodynamics can be seen as a consequence of the presence of a bifurcate Killing horizon, and the relations (57-58) between "Killing" and "inertial" times.

## VI. CONCLUSION AND FUTURE DIRECTIONS

We have derived both the Hawking and Unruh effects, considering both a Bogoliubov transformation approach between vacuua and a Wick rotation approach. A closer analysis reveals that the presence of a bifurcate Killing horizon and the resulting time translation symmetry between "Killing" and "inertial" time underlies the similarity between these two effects and their calculation approaches. Given that vacuua are determined by positive frequency modes defined with respect to a timelike Killing vector and that making time imaginary corresponds to a rotation between coordinates, a transformation between $(u, v)$ and $(U, V)$ coordinates is at the heart of both the Bogoliubov transformation and Wick rotation [27]. The "observer-dependency of particles," in the words of Gibbons [19], therefore stems from the presence of a bifurcate Killing horizon.

In future work, we would like to extend this analysis and perform a detailed quantum measurement theory treatment of an asymptotic observer measuring Hawking radiation, or an accelerating one. In particular, we would like to analyze the Unruh-DeWitt detector formalism, which provides a particle-in-a-box model of a detector with response function to be excited to energy $E$ given by

$$P(E) = |\langle E|m(0)|0\rangle|^2 \int_{\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-iE(\tau-\tau')} \langle\psi|\phi(x(\tau))\phi(x(\tau'))|\psi\rangle \tag{75}$$

where $\psi$ is the detector ground state and $m$ is the coupling between the field $\phi$ and the detector. Using the modern language of POVMs (Positive Operator Value Measures) [28], we might be able to define measurable observables for the Unruh-DeWitt detector and perhaps analyze the quantum fisher information, which quantifies how well one can estimate a certain parameter such as the temperature [29].

Moreover, the Bogolioubov transformations which indicate the presence of particle production in the Hawking effect are mathematically the same as transformations which govern the production of squeezed states of light from electromagnetic wave mixing processes in nonlinear crystals. The squeezing operator across two modes of the electromagnetic field is given by

$$S(\xi) = \exp\Big(\xi^* ab - \xi a^\dagger b^\dagger\Big) \tag{76}$$

which is nearly identical to the unitary transformation associated with the Bogolioubuv transformations between in and out Unruh vacuums [19]

$$U = \exp\Big(\gamma_{ij} b_i b_j - \bar{\gamma}_{ij} b_i^\dagger b_j^\dagger\Big) \tag{77}$$

Thus, recent work has considered interpreting the black hole as a producer of squeezed states, which are highly entangled just like the Hartle Hawking vacuum state [30]. We would like to explore this connection to quantum optics further.

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